Managing Fertilizer Programs on Leased Land in the Eastern Corn Belt

prepared for and presented at:
Indiana Crop Advisor Conference
December 16-17, 2003
(revised December 19, 2003)

Terry L. Kastens, Ph.D. agricultural economist, Kansas State University (tkastens@ksu.edu)
Kevin C. Dhuyvetter, Ph.D. agricultural economist, Kansas State University (kcd@ksu.edu)
John P. Schmidt, Ph.D. soil scientist, Kansas State University (schmidt@ksu.edu)

Introduction

Farm owners, operators, managers, and related decision makers such as crop consultants are becoming more aware of the *capital* nature of soil fertility, meaning that a number of annual crop input expenditures are more accurately characterized as multi-year investments. Likely, this has long been understood in the case of lime applications and soil pH, but not as widely understood in the case of P and K. Also, increased focus on build/maintain/drawdown fertility programs relative to annual sufficiency programs likely has increased decision makers' interest in treating crop inputs as capital investments.

The capital nature of fertilizer expenditures means that a portion of such expenditures likely are capitalized into land values and rents, which leads to profit opportunities for those possessing a better understanding of how such investments are valued. In short, holding other land characteristics constant, tracts or farms with higher fertility will be valued more highly, with lower values assigned to tracts or farms with lower fertility. Naturally, the relevant questions focus on *How much higher or lower?*

With rented land, capital treatment of crop inputs has an added dimension in that operators (tenants) may not control the land for a sufficiently long time period to recoup their crop input investments. This means that tenants may desire compensation from landlords for unused portions of their investments at the time of lease termination. Similarly, landlords may wish to extract penalties from tenants who inappropriately "draw down" soil fertility during the leasing period. Or, they may impose constraints on tenants related to minimum fertilizer application rates in order to protect the landlord's investment in a "good" historical fertilizer program. Regardless, profit opportunities arise for those decision makers with a better understanding of the economics of crop input investment.

The problems and opportunities associated with capital treatment of crop inputs arise in both cash rental arrangements and in crop share arrangements. An understanding around one type of arrangement leads immediately to an understanding around the other type. Consequently, this paper and related research considers only the simpler of the two arrangements – cash rent. But, it is easy to translate what is learned here to share renting situations. To foster this translation, the reader is directed to the Dhuyvetter and Kastens paper and related Excel-based spreadsheet, *KSU-Lease*, which can be retrieved from the website www.agmanager.info.

With some interpretation as described later, this research relies heavily upon the tri-state (Michigan, Ohio, and Indiana) fertilizer recommendations (TSFR), compiled by scientists at the three associated land grant universities (Vitosh, Johnson, and Mengel, 1995). Additionally, we draw from personal conversations with Kim Polizotto, agronomist for PotashCorp/PCS Sales, and with Tony Vyn, extension agronomist at Purdue. But, neither of these agronomists has had the opportunity to review our research. Consequently, any conclusions we reach or inferences drawn are our own.

This type of research relies heavily upon agronomic and economic assumptions. We recognize that the most appropriate assumptions (especially the agronomic ones) for one geographical area are different from those of another area. Nonetheless, to enhance understanding where needed, we have tried to make assumptions believed relevant for a major part of the area's corn and soybean production. Regardless, it should be remembered that the underlying theoretical framework and disciplinary principles likely do not change much across different geographical areas. Thus, the analysis and results put forth here easily can be modified to accommodate other geographical areas.

This paper begins with a discussion of time value of money concepts. Then, the formulas underlying the TSFR fertilizer recommendations are discussed. Following a statement about crop prices and crop input prices (crop inputs are N, P, and K fertilizer, and lime), a framework is put forth for calculating rent premiums associated with soils with high levels of fertility. Because this framework leaves a number of important unanswered questions, we next present a thread of research we are just beginning, which is designed to transform TSFR's fertilizer recommendations (or those from any other source) into a mathematical yield model that can be used to answer important economic questions related to fertilizer and lime investment. That section of the paper is somewhat laborious to read. However, we wanted to provide some documentation, partly for ourselves to guide continuing research, but also for those who might help in an ongoing dialogue around soil fertility investment issues. Our future research in this area undoubtedly will be shored up with appropriate academic references to relevant research that has gone before. In the meanwhile, we place our ideas and analysis on the table for consideration by agronomists, economists, and farm business managers who may have an interest.

Time Value of Money

Invested money always has an opportunity cost – because it always can be invested elsewhere to earn a rate of return. For example, it might be placed in an interest-bearing bank account. Or, it might be used to pay off debt instead, ultimately saving interest rather than earning it. Because competing investments often have expected returns that vary in magnitude and/or in timing, investment evaluation must consider the time value of money. In its simplest terms, a dollar received today is not the same as a dollar received next year – because the dollar today can earn interest. Consequently, obtaining a rudimentary understanding of the time value of money (referred to as net present value analysis in the financial and economic world, or NPV) is critical for acquiring an understanding of fertilizer as a capital investment.

In an NPV framework, the value n years in the future (V_n) of a value invested today (V_0 – today is depicted as year 0), with interest reinvested, and given an interest rate i (a percentage expressed as a decimal) can be depicted as:

$$V_n = V_0 * \left(1+i\right)^n \; . \qquad [1]$$

The relationship shown in Equation [1] also can be used to answer the question, What would be the value today of an amount of money expected to be received n years in the future? In that case, Equation [1] is algebraically reordered or solved for V_0:

$$V_0 = \frac{V_n}{\left(1+i\right)^n}\,. \tag{2}$$

Equation [2] states that, given an interest rate of 8% (i.e., 0.08), a person would be indifferent between receiving \$100 thirty years in the future and receiving \$9.94 today. That is because $100/(1.08)^{30}$ is \$9.94. Put another way, an investment of \$9.94 earning an annual interest of 8% a year (assuming the interest is reinvested each year) would grow to exactly \$100 in 30 years. Equation [2] depicts a process known as discounting. The value V_n is said to be discounted by a discount factor equal to $1/(1+ i)^{n}$, to arrive at today's value V_0. We use *dfac* to represent the term $1/(1+i)$, so that the discount factor associated with Equation [2] is $dfac^{n}$.

For businesses, and farms in particular, income taxes are paid against net earnings. Put another way, expenses are deducted from gross income to become taxable income. We assume that decision makers want to maximize after-tax profits not pre-tax profits (because only after-tax profits can be used for consumption or for additional investment). That means it is most appropriate to treat V_0, V_n, and i as after-tax values. For farms (which often are sole proprietorships), we start with a typical borrowing rate on agricultural loans at banks, and then reduce that rate to account for typical or average income tax rates (i.e., federal income tax, state income tax, and self-employment tax). Using *tax* to represent the sum of the relevant tax rates, we have

$$i = observed\ bank\ interest\ rate * \left(1 - tax\right). \tag{3}$$

In practice, after-tax values often must be transformed back to pre-tax ones because managers are more used to comparing pre-tax values. For example, when it is acknowledged that "the going cash rent is \$120/acre," this is a pre-tax number. Because cash rent is a tax-deductible expense, assuming a tax rate of 0.40, the after-tax cash rent would be \$72/acre. Mathematically, in Equation [2], once we assume an after-tax interest rate, it is irrelevant whether we start with an after-tax V_n, ending up with an after-tax V_0 that is subsequently converted back to its pre-tax equivalent (by dividing by 1&*tax*), or simply start with a pre-tax specification of V_n and then merely use the computed V_0 value (which will be a pre-tax number). The important thing is to use an after-tax discount rate, i.e., an after-tax *i*.

NPV analysis always considers some finite or infinite time horizon. As long as the stream of monies can be represented by a constant value, then we are interested in other constant discount terms that depend on *dfac* and the time horizon *n* (typically dimensioned as years). For example, consider a constant annual amount of money $\$K$, that will be received each year starting one year from today and ending *n* years from today. Using the mathematics of numerical sequences, the relevant discount term ($DFAC_n$) is

$$DFAC_n = \frac{dfac - dfac^{n+1}}{1 - dfac}\,. \tag{4}$$

That is, today's value of the *n* future annual receipts of $\$K$ is $\$K$ ($dfac^1$ + $\$K$ ($dfac^2$ + . . . + $\$K$ ($dfac^{n}$ = $\$K$ (($dfac^1$ + $dfac^2$ + . . . + $dfac^{n}$). Moreover, using the mathematics of numerical sequences for the parenthetical part, this reduces simply to $\$K$ ($DFAC_n$. Putting some numbers to the expressions, consider the following. Assuming 20 future receipts (n = 20) a bank interest rate of 8% (i.e., 0.08) and a tax rate of 40% (i.e., *tax* = 0.40), results in *i* = 0.08 ((1&0.40) = 0.048. Then *dfac* = 1/1.048 = 0.9542, and $DFAC_{20}$ = (0.9542&0.9542^{21})/(1&0.9542) = 12.6765. Suppose an investment were made today that resulted in higher crop yields for each of the next 20 years (and only 20 years), where the higher crop yield each year, when taken times the crop price, resulted in $10/acre. How much could be paid for that investment today? From our example, the number is 12.6765 ($10 = $126.765/acre. Finally, it should be noted that the *DFAC* associated with an infinite horizon is simply *dfac*/(1&*dfac*). For the after-tax interest rate of 0.048, $DFAC_4$ = 20.8341.

Equally as relevant, the process described in the preceding paragraph could be reversed. That is, we might ask, What would be the annual revenue increase needed over the next *n* years to "pay off" an investment of $Q today? This is a standard loan amortization problem and so it is better to characterize it in terms of *i* rather than in terms of *dfac*. Given an *i*, the amortization factor associated with *n* equal payments starting one year from today and ending *n* years from today, referred to here as $AMORT_n$, is

$$AMORT_n = \frac{i}{1-(1+i)^{-n}} . \tag{5}$$

For the example just described, *n* = 20 and *i* = 0.048, which implies that the amortization factor is 0.048/[1&(1.048)$^{-20}$] = 0.048/(1&0.3915) = 0.0789. That is, for every $1 invested today, the annual "payment" would be $0.0789. If the initial investment were the same as that noted earlier (i.e., $126.75/acre), the annual "payment" would be 0.0789 (126.75 = $10/acre, which gives us back the result we started with in the original problem. Finally, the infinite-horizon value resulting from Equation [5] is merely the interest rate *i* itself.

In general, either the finite- or infinite-horizon version of either Equation [4] or Equation [5] can be used to appropriately characterize and quantify the relationships that capitalize crop input expenditures into land values and rents. In the context of the preceding examples, consider that the $126.75/acre may represent the added present value associated with land parcel A over parcel B because parcel A had higher fertility. In that case, if the land were rented for 20 years, parcel A should garner up to $10/acre higher annual cash rent than parcel B. Rented into infinity, the higher rent would be $6.08/acre annually, which is simply 126.75 (0.048, because $AMORT_4$ = *i* = 0.48.

Tri-state Fertilizer Recommendations (TSFR)

Before proceeding, it is worthwhile to recount the TSFR formulas for the major elements of interest, N, P, K and lime, and for only the two major crops of interest in the area, corn and soybean. To assist in our example, we use somewhat different notation than that used by TSFR. Negative recommended rates from any of the formulas are assumed to be 0.

N recommendations for corn

For corn, the nitrogen fertilizer recommended rate in lb/acre (*Nrec*) is specified as

$$Nrec = 1.36 * yield\ potential - 27 - N\ credit\ , \tag{6}$$

where *yield potential* (sometimes referred to as yield goal) is taken to be 10% above expected or average yield, and where *N credit* is taken to be 30, which is the credit that TSFR assigns to situations where corn follows soybean. For our purposes, we restate Equation [6] as

$$Nrec = 1.36 * YG - 57\ . \qquad [7]$$

The build/maintain/drawdown framework

TSFR treats P and K in the following framework. Below some critical soil test level (*CL1*), besides crop removal rates, an additional amount of fertilizer is recommended that is designed to bring soils to the *CL1* level in four years. Within some soil test range above the *CL1* level, a maintenance amount of fertilizer is recommended so that soil test levels can be maintained at a constant level. Within the maintenance region, no yield response to fertilizer is expected. Thus, while in this range, it is suggested that managers can make multi-year applications with no fear of yield reductions.

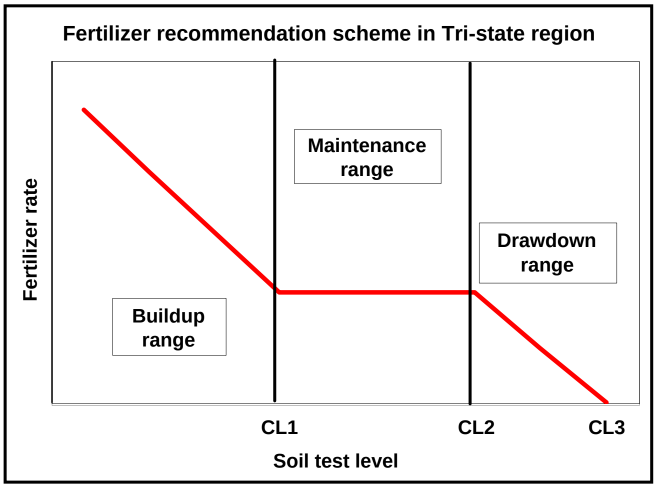

Figure 1

Although TSFR characterizes maintenance as a constant crop removal (*CR*) rate (lb fertilizer per bu) times *yield potential*, we use *expected* yield against TSFR's removal rates. Otherwise, given that *yield potential* likely is higher than expected yield, simulated soil tests would rise over time rather than stay constant. Of course, TSFR may have been thinking about a tranformational inefficiency that requires application of an amount of fertilizer greater than crop removal to sustain soil test. But, that would mean we would need to use a different *effective* transformation rate than implied by TSFR in our mathematical models to keep simulated soil tests from rising over time. Either way, our results would be the same, i.e., we believe TSFR does not intend for soil tests to rise over time once they are above certain critical levels. TSFR's formulas imply that the maintenance range is capped at another critical level, which we refer to as *CL2*. At soil test levels above *CL2*, drawdown is suggested, where less than crop removal fertilizer rates are recommended so that residual soil nutrients can be used. Finally, at soil test levels above some sufficiently high soil test level (we call it *CL3*), zero fertilizer is recommended. Figure 1 shows a schematic of this framework that is similar to the one shown in the TSFR publication.

Recommended P rates (*Prec*) are in terms of lb/acre of phosphate (P_2O_5) and are based on soil test P in ppm (*STP*) using the Bray P1 test. Recommended K rates (*Krec*) are in terms of lb/acre of potash (K_2O) and are based on soil test K in ppm (*STK*) using the 1 N ammonium acetate test to estimate K availability. Using our notation, and with *EY* representing expected yield rather than *yield potential* as suggested by TSFR, the buildup equations for P and K are as follows:

$$\text{Buildup P and K recommendations (for corn and soybean; } CR \text{ \& } CL1 \text{ crop-specific)}:$$

$$Prec = (CL1 - STP) * 5 + EY * CR\ , \qquad [8]$$

$$Krec = (CL1 - STK) * (1 + 0.05 * CEC) + EY * CR + 20\ .$$

In Equation [8], *CL1* values are crop-specific and crop removal rates (*CR*) also are crop-specific. Note that, besides depending on *STK*, TSFR's *Krec* depends upon a measure of cation exchange capacity

(*CEC*), where *CEC* units are meq/100g. The maintenance equations for P and K are as follows:

$$\begin{aligned}&\text{Maintenance P and K recommendations (for corn and soybean; } CR \text{ \& } CL1 \text{ crop-specific):}\\ &Prec = EY * CR\,, && [9]\\ &Krec = EY * CR + 20\ .\end{aligned}$$

The drawdown equations for P and K are as follows:

$$\begin{aligned}&\text{Drawdown P and K recommendations (for corn and soybean; } CR \text{ \& } CL1 \text{ crop-specific):}\\ &Prec = \left(EY * CR\right) * \left(1 - \frac{STP - \left[CL1 + 15\right]}{10}\right), && [10]\\ &Krec = \left(EY * CR + 20\right) * \left(1 - \frac{STK - \left[CL1 + 30\right]}{20}\right).\end{aligned}$$

According to TSFR, *CL1* for P is 15 ppm for both corn and soybean. For P, *CL2*, the end of the maintenance range, is *CL1*+15, which is 30 ppm. Thus, the maintenance region is 15 to 30 ppm for P. Based on the *Prec* formula of Equation [10], when *STP* = 40, the rightmost term becomes 0 and the recommended P rate is 0. Thus, the third critical level, *CL3*, is 40 ppm for P. For K, the end of the maintenance range (*CL2*) is 30 ppm above *CL1*. And, *CL3* = *CL1*+50, the point where recommended K rates become 0. However, unlike with P, the *CL1* level for K is not fixed. Rather, it depends on *CEC*:

$$\text{for K only: } CL1 = 75 + 2.5 * CEC\ . \qquad [11]$$

Crop removal fertilizer rates are an important part of each of the recommendations shown in each of the equations above. TSFR assumes crops remove 0.37 and 0.80 lb P_2O_5 per bushel of corn and soybean, respectively. K_2O removal rates are 0.27 and 1.40 lb per bushel of corn and soybean, respectively. Additionally, TSFR assumes it takes 20 lb of excess (above crop removal) P_2O_5 to increase *STP* by 1 ppm. That can be seen by the fact that the *Prec* formula in Equation [8] multiplies the desired change in *STP* by 5, along with the assumption that buildup occurs in 4 years. Similarly, the *Krec* formula of Equation [8] also implies an excess-K_2O-to-*STK* transformation rate. However, this transformation rate is not constant as it is for P, rather it depends on *CEC*. For example, at *CEC* = 15, the transformation rate is 7, which is (1+0.05 (15) (4. Thus, the transformation rate for K only is

$$transformation\ rate = \left(1 + 0.05 * CEC\right) * 4\ . \qquad [12]$$

Lime recommendations

TSFR suggests that the optimal *soil pH* level for both corn and soybean is 6.0. Because Vyn suggested 6.1, and because targeted soil pH in a less-than-annual liming sequence probably would be slightly higher than the optimal level, we use 6.1 for the targeted soil pH in this research. TSFR bases its recommended lime rate on the SMP *buffer pH* test and provides formulas for liming (TSFR assumes lime has a neutralizing value of 90%) to either 6.0, 6.5, or 6.8, but not to 6.1. Hence, we devised a formula targeting 6.1 that was a combination of TSFR's 6.0 and 6.5 formulas. Our formula is

$$Limerec = 51.52 - 7.42 * buffer\ pH\ . \qquad [13]$$

TSFR suggests that lime should be applied whenever *soil pH* drops 0.2 to 0.3 pH units below the recommended level. Using the 0.3-unit suggestion, along with the 6.1 *soil pH* target, implies that it is optimal to bring a 5.8 *soil pH* up to 6.1 with lime, let it fall again to 5.8 over time, and then bring it back to 6.1, and so on. For this work, we make the simplistic assumption that *soil pH* drops 0.15 pH units per year across a corn-soybean rotation. With these assumptions, it would be recommended to lime every two years, each time bringing *soil pH* back to 6.1. We recognize that many managers likely apply lime less frequently than biannually. On the other hand, it is typically recommended that managers using no-till apply lime more frequently than those using conventional tillage practices. Thus, it could be that our assumptions are most fitting for no-tillers.

Prices and Other Economic Assumptions

Since this is a study of crop inputs as a capital investment, economic assumptions should be reasonable expectations over several years rather than over just the current year. Consequently, long-term prices and other economic values are more appropriate than short-run values.

Crop prices

We use the 10-year (1993 through 2002 crop years) average USDA-NASS (National Agricultural Statistics Service) corn and soybean prices for Indiana. However, to more closely mimic farm-level prices, we substitute the U.S. loan rate in years when the reported Indiana cash price was lower (1999 for corn and 1998 through 2001 for soybean). The end result is that we assume a \$2.38/bu price for corn and a \$5.91/bu price for soybean.

Fertilizer and lime prices

We use the 5-year (1998 through 2002 calendar years) average USDA-NASS fertilizer prices. For N, we assume that 45% of all N fertilizer is applied as anhydrous ammonia (82% N), 35% as urea (45% N), and 20% as UAN (32% N). Using these product weights, the average fertilizer N price is \$0.21/lb of N. For P, we assume that 25% of all P fertilizer is 10-34-0 and 75% is 18-46-0. After accounting for the fertilizer N value in these products, the average fertilizer P price is \$0.22/lb of P_2O_5. For K, we use the price series for muriate of potash (60% K_2O), resulting in an average price of \$0.13/lb of K_2O.

Lime prices do not have a comparable generalizing historical price series like P and K. So, after checking with a couple of crop service providers, we arbitrarily assume a lime price (90% neutralizing) of \$14.75/T, which also includes the application fee. However, we also assume a minimum charge of \$14.75/acre, which assumes that providers will only offer their services for \$14.75/T if they apply at least 1 T.

Based on personal contacts and Doanes 2003 Custom Rates, and where needed, we assume that P and K fertilizer are associated with a \$4.80/acre application fee. We assume that N is associated with a

\$6.00/acre application fee, which corresponds to an NH_3 application.

Interest and tax rates

We assume a bank interest rate of 8% and an income tax rate of 40%, where the income tax rate is assumed to include federal income taxes, state income taxes, and self-employment taxes.

Expected crop yields

Expected crop yields are an important component to any economic analysis of fertilizer investments. Based on conversations with Polizotto and Vyn, we believe that historical average USDA-NASS crop yields in Indiana are too low to be reasonable representations of many progressive farmers' yields. Consequently, after considering information from NASS, Polizotto, and Vyn, we selected a corn yield of 165 bu/acre and a soybean yield of 50 bu/acre.

Basic Long-run Implications of TSFR Recommendations and Land Rental Rates

It should be noted that the TSFR recommendations are in large part an oversimplification of the underlying agronomic and economic theory. For example, TSFR assumes no corn or soybean yield response to either fertilizer P or soil test P above $STP = 15$, at least in the year of application. For now, we mostly will ignore any potential mathematical, economic, and agronomic inconsistencies that may arise from such an oversimplification.

To examine the impact of soil fertility on rental rates we must begin with a benchmark. In most cases we will consider that benchmark to be the rental rate expected to be paid when soil test levels are at their long run optimums, and by farm managers who expect to control the land forever. In practice, decisions made over a horizon greater than about 20 years are similar to ones made over an infinite horizon. Though it is tempting to say such rental rates are also market rental rates, that presumes decision makers operate in a long-run time frame such as >20 years, which may or may not be the case. Of course, in areas where rental arrangements tend to be ongoing for many years, our benchmark rents probably are not too far from market rental rates.

The case for high P

We assume that the benchmark rental rate is one that is consistent with STP levels between $CL1$ and $CL2$. That is, given the crop (corn or soybean), TSFR recommends the same fertilizer P rate and expects the same crop yield any time STP is between 15 and 30. Above $STP = 30$, the future savings in fertilizer can be used to calculate an expected rental premium associated with such lands. Notice that such premiums will depend only on agronomic assumptions and the price of phosphate fertilizer and not on the crop price – because yields are assumed to be the same any time $STP > 15$.

Given an STP level above $CL2$ (here, $CL2 = 30$) but below $CL3$ (here, $CL3 = 40$), in Equation [10] we can see that the immediate savings of phosphate is $EY \Big(CR \Big(0.1 \Big((STP \& CL2)$. Using STP_1 to denote STP in year 1 (1 year from today, i.e., for the upcoming harvest) and $saveP_1$ to denote the year-1 savings of phosphate fertilizer in lb/acre, we have

$$saveP_1 = EY * CR * 0.1 * \left(STP_1 - CL2 \right). \qquad [14]$$

When $STP > CL3$, the fertilizer savings simply is the crop removal rate, $EY\big(CR$. From today's (year 0) standpoint, the number of lb/acre of phosphate saved calculated by Equation [14] is multiplied by the expected price of phosphate and also by the necessary discount term, $dfac^1$. Strictly speaking, we are assuming that fertilizer expenditures come the same time as crop sales, with both values discounted back to the time point of 1 year earlier. We could consider separate discount terms for crop sales and fertilizer expenditures to account for timing assumptions. Or, we might increase the expected price of fertilizer by a small amount, say $i/2$, to make them equivalent to crop sales time-wise (assumes fertilizer expenditures come 6 months ahead of crop sales). Since the results will change very little, we choose to ignore such adjustments in this part of our discussion.

Because Equation [14] assumes applying less than crop removal P, the *STP* will drop over time, asymptotically approaching *CL2* over time. Recall that TSFR assumes a phosphate-to-STP transformation rate of 20. Thus, *STP* is expected to drop by 0.05 times the fertilizer savings. Hence, the equation for *STP* in year 2, STP_2, is

$$STP_2 = STP_1 - 0.05 * saveP_1 . \qquad [15]$$

Then, the fertilizer savings for year 2, $saveP_2$, is computed using Equation [14], only replacing STP_1 with STP_2. Then, the discounted value of that savings is calculated as described above, only now using a discount term of $dfac^2$, and so on. Then, the discounted values can be summed over the *n*-year time horizon (i.e., the length of the rental contract) to result in a NPV of soil fertility. Finally, that NPV can be amortized by multiplying it by the amortization factor calculated according to Equation [5], resulting in a constant annual rental premium that can be assigned to the rental arrangement covering the time horizon of interest.

The process just described can easily be programmed in a computer spreadsheet. Furthermore, one can easily assume a corn-soybean rotation, where each crop has its own expected yield and its own crop removal assumption. For visual presentation, we use the fertilizer price (\$0.22/lb P_2O_5), interest rate (0.048), and tax rate (0.40) noted earlier, along with an expected yield of 165 bu/acre for corn and 50 bu/acre for soybean in a corn-soybean rotation. It should be noted that results are slightly different depending on whether the first year is assumed to be corn or soybean; we assumed corn. Figure 2 shows the rental premiums associated with high initial *STP* levels ("re-sampling = Yes" explained later). The annual rent premiums associated with rental contracts ranging from 2 to 20 years are shown. Four initial *STP* levels are considered, 40, 50, 60, and 70 ppm.

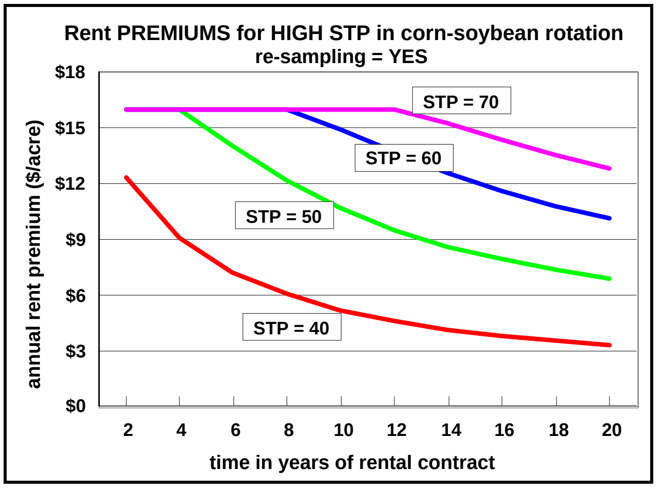

Figure 2

Figure 2 makes it clear that renting land with high *STP* has substantial value, especially over short-term rental arrangements. The horizontal line in the figure that is common to three of the four *STP* levels at some horizons is due to applying 0 fertilizer, which saves not only crop removal phosphate but also the $4.80/acre application charge associated with 0 fertilizer. It should be noted that, if a manager chooses to capitalize high fertility over a short time period by paying a higher rent, he may be faced with the possibility that he will have to "talk his landlord" down in terms of rental rates after the contract ends because of the lower *STP* levels observed at that time – assuming he wishes to renew his rental arrangement – and, landlords probably are reluctant to accept lower rents when they are accustomed to rents that rise over time. Hence, the tenant may want to consider a rent premium appropriate for a longer time horizon even if the initial contract is a short-term one. Also, although we normally recommend using long-term fertilizer prices in rent premium analysis, if a manager is really sure that a rental arrangement is going to be short, say 2 years, then it might be appropriate to use current fertilizer prices. The idea is that, if a land parcel that is especially high in *STP* comes up for rent in the face of high fertilizer prices and a short-term contract, it would be especially advantageous to pay a premium for such land. In the figure, 50% higher phosphate prices and 50% higher application charges would make all premiums exactly 50% higher than those shown.

Notice that the preceding framework expressed in Equations [14] and [15] implicitly assumes that *STP* is re-sampled each year, with a new fertilizer savings calculated each year. Since the application rate is crop removal phosphate less the fertilizer savings each year, this means that the application rate will change each year. An alternative assumption could be made, namely that of no re-sampling. In that case, the manager is assumed to calculate a recommended fertilizer rate based off of the initial *STP* and the *CL2* value. For example, for any initial *STP* > 40, the recommended rate is 0 phosphate. Then, the manager is assumed to apply this rate (0, if *STP* > 40) each year over the number of years (based on the formulas) it will take until soil fertility drops to *CL2*. Figure 3 displays the same results as Figure 2, only assuming no re-sampling.

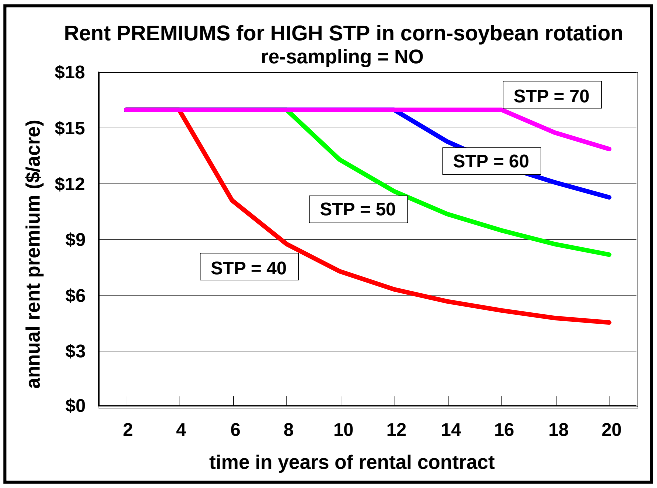

Figure 3

Clearly, the results of Figure 3 are different from those of Figure 2, especially for the *STP* = 40 line. That is partly because Figure 2 (re-sampling) assumed that only one year (the first year) in any of the horizons associated with *STP* = 40 had a 0 fertilizer rate. Further, that meant only one year benefitted from saving the $4.80/acre application charge. On the other hand, Figure 3 assumed that 0 fertilizer was applied for four years in the case of *STP* = 40 (i.e., until *STP* dropped below 30). TSFR was not exactly clear about what was intended regarding re-sampling. But, one apparent inconsistency would seem to arise. That is, a field with an initial *STP* level of 35 ppm would be recommended a phosphate rate of 50% of crop removal. Yet, another field, which happened to be observed at an *STP* level of 35 ppm while it was on its way down after starting at a level above 40 ppm would be recommended a phosphate rate of 0 lb/acre assuming no re-sampling. Given this inconsistency, it seems that TSFR would want to err on the side of soil sampling. If so, Figure 3 would be a more reliable rendition of the expected rent premiums than Figure 2.

The case for low P

It seems highly implausible that a short-term renter would follow the 4-year build program suggested by TSFR. In particular, why would he want to continue building *STP* shortly before the land is turned over to someone else who would benefit from his expenditures? In short, TSFR never makes it clear over what time horizon a 4-year build is expected to be appropriate. Would it be appropriate for a land tenure period of 6 years? Of 8 years? Perhaps the most reasonable answer is that it would be appropriate for a land *owner* rather than a tenant, that is, a manager with a long horizon, say 20 to 30 years. But, even assuming that, we still do not know how to guide the short-term renter using the TSFR recommendations. Hence, determining rental discounts that should be applied to land with low *STP* is not straightforward. For that matter, the same goes for K and soil pH. Consequently, we defer a discussion regarding rental discounts associated with low fertility until later in this paper.

High K

Adjusting rental rates for high *STK* proceeds in a similar manner to the calculations for high *STP* already described, only with using the proper TSFR buildup and drawdown equations. Second, the critical level *CL1* is not fixed as it is with P, rather it depends on *CEC* as in Equation [11]. Then, the end of the maintenance range, *CL2*, is always 30 ppm above *CL1*, and the no-fertilizer point (*CL3*) begins at 20 ppm (i.e., *CL3* = *CL2* + 20 = *CL1* +50). Next, rather than using a constant fertilizer-to-soil-test transformation rate as we did for P, the transformation rate for K depends on *CEC* as in Equation [12]. As with P, the transformation rate is used to calculate expected *STK* next year based on the amount of savings in potash fertilizer.

The fertilizer savings formula for the case of high *STK* (i.e., *CL2* < *STK* < *CL3*) is

$$saveK_1 = \left(EY * CR + 20\right) * 0.5 * \left(STK_1 - CL2\right). \qquad [16]$$

As with *STP*, for *STK* > *CL3*, the fertilizer savings simply is crop removal K, here $EY\left(CR\right.$ + 20. The formula for adjusting next year's expected *STK* based on this year's *STK* is

$$STK_2 = STK_1 - \frac{saveK_1}{\left(1 + 0.05 * CEC\right) * 4}. \qquad [17]$$

For the graphical representations we first assume a *CEC* value of 12 meq/100g. This implies a *CL1* value of 105 ppm and a *CL2* value of 135 ppm. We consider the same rental contract lengths considered in the P assessment, and the fertilizer price (\$0.13/lb K_2O), interest rate (0.048), tax rate (0.40), and expected yields (corn 165 bu/acre; soybean 50 bu/acre) assumed earlier. Finally, we show only the situations that consider re-sampling of soil over time. Figure 4 shows the rent premiums associated with high *STK*, considering initial *STK* levels of 155, 175, 195, and 215 ppm. The premiums would be calculated against a benchmark cash rent for land that has *CEC* = 12 and where *CL2* < *STK* < *CL3* (here, 105 < *STK* < 135).

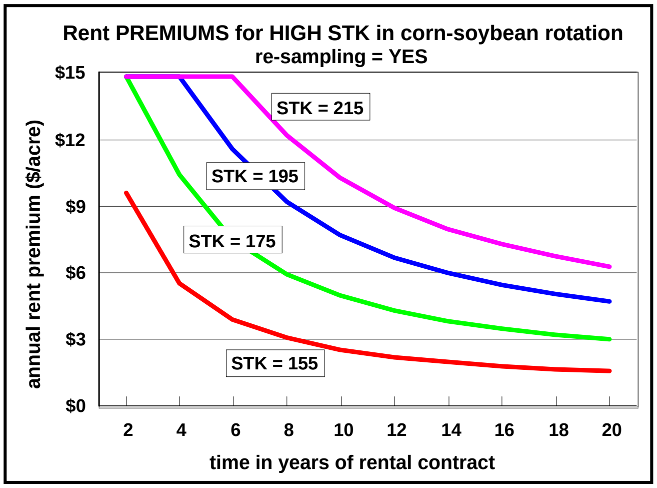

Figure 4

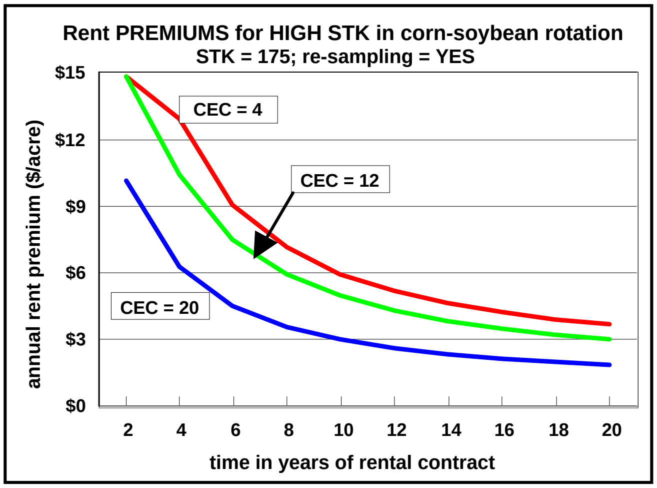

Figure 5

TSFR reports that high clay (hence high *CEC*) soils require higher K levels to support optimum crop growth, which is the reason that critical levels for K are based on *CEC*. In this setting, higher *STK* levels are especially valuable in the face of low *CEC*, because such high *STK* levels are further above the critical levels since the critical levels are lower for lower-*CEC* soils (implying greater fertilizer savings). Figure 5 shows the rent premiums associated with a single high *STK* level (*STK* = 175), only for three different levels of *CEC* (4, 12, and 20). Underlying the figure, the benchmark rent would vary by *CEC* (because *CL1* and *CL2* vary by *CEC*), and would be associated with land where *CL1* < *STK* < *CL2*. The basic presumption is that market cash rents probably adjust to the prevailing *CEC* in an area, hence to the prevailing K maintenance range.

Characterizing pH to get at rent premiums for different soil pH

TSFR's lime recommendations are principally physical ones, answering the question, How much lime does it take to reach a targeted *soil pH* given an SMP *buffer pH* level. As noted earlier, TSFR references to the economics of liming are weak, with about the only statement being that lime should be reapplied when *soil pH* drops 0.2 to 0.3 pH units below the recommended level of *soil pH*. In particular, TSFR made no reference about how much *soil pH* is expected to drop over time. For that, we noted earlier that *soil pH* might drop 0.15 units per year. We use that simplistic assumption to guide evolution of *soil pH* over time in the absence of liming. Further, recall that earlier we had assumed that the optimum level was a *soil pH* of 6.1 rather than the 6.0 stated by TSFR for corn and soybean.

As suggested earlier, in the long run, optimal lime rates are expected to be consistent with a sequence of liming to *soil pH* = 6.1, letting *soil pH* drop to 5.8 over the next two years, and subsequently liming back to *soil pH* = 6.1. *Soil pH* levels are observed and a *soil pH* level is what is targeted with lime applications. Yet, the amount of lime needed is based on *buffer pH*. Thus, what is needed to continue the analysis is some mathematical relationship that gives reasonable expectations of *buffer pH* from known or simulated values of *soil pH* and possibly other factors. We consider that "other factor" to be *CEC*, with the idea that high *CEC* values are associated with strongly buffered soils and result in lower *buffer pH* values relative to low-*CEC* soils.

In simulating *buffer pH* levels from *soil pH* levels, we make the arbitrary assumption that the minimum *buffer pH* that can be observed is something greater than the *soil pH*, supporting the idea that adding a buffering agent with a pH of 7.5 (the pH of the SMP buffering agent) to soil would always "pull up" the *soil pH* somewhat, even in the case of highly buffered soils. Put another way, we assume that the most a soil can "pull down" the pH from the SMP buffering agent pH of 7.5, is to a level of *min buffer pH*, which will depend on the original *soil pH* level. Furthermore, we make the additional assumption that the highest *buffer pH* that might be observed is 7.0. This assumption is because lime recommendation tables from various sources nearly never show *buffer pH* levels above 7.0, and even rarely above about 6.8. We arbitrarily define *min buffer pH* as

$$min\,buffer\ pH = 7.0 * 0.25 + soil\ pH * 0.75\,, \qquad [18]$$

which defines the *min buffer pH* to be the *soil pH* level, plus 25% of the range between 7.0 and the *soil pH* level, and where *soil pH* > 7.0, *min buffer pH* is assumed to equal 7.0. Next, we arbitrarily presume an association between *buffer pH* and *CEC*:

$$buffer\ pH = 7.0 * \left(1 - \frac{CEC}{maxCEC}\right) + min\,buffer\ pH * \left(\frac{CEC}{maxCEC}\right), \qquad [19]$$

where *maxCEC* is taken to be 24. In Equation [19] it can be seen that *buffer pH* is placed proportionately between *min buffer pH* and 7.0, depending on the level of *CEC*. Likely, Equation [19] does not describe the exact relationship between *CEC* and *buffer pH*. Nor does it have to, since *buffer pH* is determined by laboratory measurement and it is that measurement which will determine lime recommendations. The specification was chosen to reasonably balance subjective information from Vyn and Polizotto with TSFR information. In short, Equation [19] is designed to allow for some simulation calculations later in this paper. Moreover, it does capture one important salient feature about the relationship, namely that higher *CEC* soils are strongly buffered, leading to lower *buffer pH* and higher suggested lime rates.

The implications for rents of high soil pH

As in the P and K case, the relevant question is, What are the lime savings associated with high *soil pH* values? The benchmark rent is considered to be that for land with an initial (at planting time) *soil pH* of 6.1. Such land presumably would be limed some time after the second future harvest and before the third crop being planted, and so on. Thus, we assume that such expenditures are attached to the time period of the second harvest, the fourth harvest, and so on. Using the lime price noted earlier of $14.75/T, and the *dfac* also described earlier (0.9542), the NPV of lime costs was calculated for the benchmark situation and rental arrangements 2 years in length (i.e., 2 crops), 4 years, and so on, through 20 years in length. As before, corn was assumed raised in odd-numbered years and soybean in even-numbered years.

To aid understanding, it might be helpful to more carefully describe the NPV analysis for the benchmark situation. A manager observes *soil pH* = 6.1 at the end of year 0 (which is the beginning of year 1, say prior to planting). So, no lime is applied in year 0 and also none after harvest in year 1 since *soil pH* will then be 5.95. However, by the end of year 2, observed *soil pH* is 5.8, so lime is applied. Using Equation [19] with an assumption of *CEC* = 12 estimates a *buffer pH* level of 6.55 associated with the *soil pH* level of 5.8. Then, using Equation [13], the lime application rate is 2.919

T/acre at a total cost of $43.06/acre. Multiplying this value by $dfac^2$ results in a discounted (back to year 0) value of $39.20/acre. Year 0 and year 1 each had 0 lime applied, so the 2-crop NPV is $0+$0+$39.20, which is simply $39.20. Then, this value could be amortized over the 2 crops or 2 years or 2 rent payment time periods using Equation [5] with $i = 0.048$ and $n = 2$. The resultant annually amortized "payment" is $21.02/acre. Since the benchmark is nothing more than a repeated sequence of every-other-year lime expenditures, each one identical, the annually amortized "payment" will always be $21.02 no matter what even-number-of-years time horizon is considered. Hence, this *is* the infinite-horizon benchmark desired. Then, the benchmark NPV or benchmark amortized values can be compared with other situations where initial *soil pH* is greater than 6.1.

Now, consider a situation involving high *soil pH*, say 6.9. In this scenario, *soil pH* does not drop below 5.8 until the end of the 8th crop or year. At that time, the expected *soil pH* value is 5.7, the *buffer pH* value is 6.5125, calling for a lime application of 3.1973 T/acre, bringing the *soil pH* back to 6.1. This begins the usual biannual application presumed for the entire benchmark series, where lime will next be applied following the 10th crop (2.919 T/acre), then after the 12th crop, and so on. The NPV of this scenario can be computed for each time horizon of interest. Obviously, each of the 2-, 4-, and 6-year horizons will have an NPV of $0 since no lime was applied until the end of year 8. Finally, at any horizon of interest, this NPV can be subtracted from the same-horizon benchmark NPV, and subsequently amortized, to result in an annual benefit (hence rent premium) associated with the high initial *soil pH* level of 6.9.

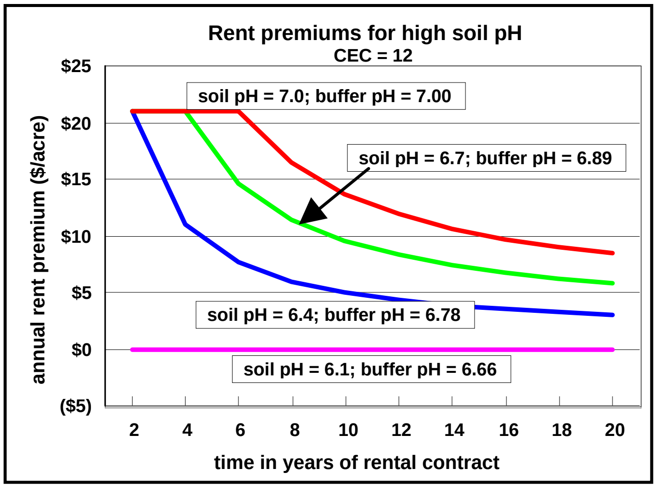

Figure 6

Figure 6 shows the rental premiums associated with initial *soil pH* values ranging from 6.1 to 7.0 (*buffer pH* from 6.66 to 7.00) and all of the assumptions already noted. The *soil pH* = 6.1 benchmark is clearly shown as a horizontal line at $0 premium. The figure shows that high *soil pH* should be associated with substantial premiums to rents, especially for short-term rental contracts. This should not be surprising knowing that such situations often would require no lime until some time after the rental arrangement had ended. It should be noted that, in our analysis, no allowance was made for reduced yields or reduced efficacy of fertilizer or herbicide that might occur with especially high levels of *soil pH*. Also, in the figure, although rent premiums diminish with longer time horizons, they do *not* become 0 with an infinite time horizon. Rather, they asymptotically approach some non-0 value. For example, the 7.0 *soil pH* line in the figure asymptotically approaches a rent premium of $5.15/acre with ever greater horizons. This implies that the lime saved in only the early years (no lime was needed until year 8) should be associated with a $5.15/acre annual premium even if the land is rented forever.

As with the K analysis earlier, it is interesting to see the possible impact of *CEC* on rental premiums via the calculated *buffer pH* values. Of course, as already noted, our depiction of the relationship between *buffer pH* and *CEC* may not be especially accurate quantitatively, but it is certainly reasonable qualitatively. Further, *CEC* is the type of variable that might be easy to proxy with automated spatially dense sensors such as those from soil conductivity measuring machines. That means the more we learn about such variables the more likely site-specific input applications will become feasible. To that end, Figure 7 shows the rent premiums associated with one high level of *soil pH*, 7.0, but at three different levels of *CEC*, thus three different levels of *buffer pH*.

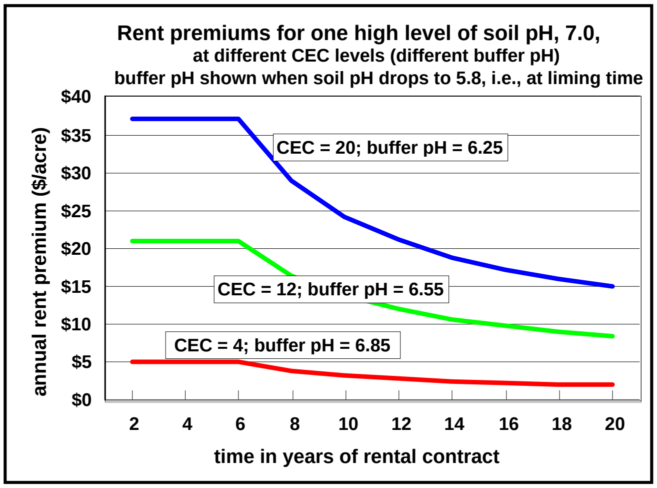

Figure 7

In Figure 6, we showed the *buffer pH* levels associated with the stated *soil pH* levels given a constant *CEC* level of 12. At *soil pH* = 7.0 (what is assumed in Figure 7), Equations [18] and [19] suggest a *buffer pH* level of 7.0 regardless of *CEC* level. Hence, different *CEC* levels are not expected to impact *buffer pH* when soil *pH* is at 7.0. But, the relevant issue here regards the *buffer pH* level expected *at the time of liming*, that is, when *soil pH* = 5.8. Hence, that is what is shown in Figure 7 across different *CEC* levels. More highly buffered soils (higher *CEC*, lower *buffer pH*) are associated with greater lime requirements at the time of liming (to bring *soil pH* back to 6.1). Consequently, renting land with high *soil pH* (here, 7.0) *and* high *CEC* (low *buffer pH*) is especially valuable since lime requirements are so high when liming is required, meaning a substantial amount of lime is “saved” when renting such land.

Extension to the Analysis: Answering Other Interesting Questions

The preceding analysis of rent premiums associated with soil fertility, even given that the various assumptions are indeed reasonable, still leaves a number of important unanswered questions. Perhaps most importantly, how should rents be adjusted in the face of *low* fertility rather than high fertility? For example, regarding land with especially low *soil pH* and a short-term rental contract, might it pay to lime to a *soil pH* less than 6.1? That is, consider the tenant who finds himself with a known 3-year lease and a *soil pH* level of 5.2. Does he really need to lime to 6.1? Or, would he be better off not liming at all, or perhaps to a lower level than 6.1? Moreover, the same questions arise around P and K. Secondly, is yield response to soil fertility really 0 above stated critical levels? Or, might there also be a slight yield benefit to especially high fertility. That would mean that the rental premiums displayed in the preceding figures understate the true premiums. Thirdly, the TSFR recommendations do not account for different crop prices. For long run decisions, this probably is not a relevant issue. But, for short-term rental arrangements, prices might have a large impact. These types of questions cannot be answered using the framework already discussed. Yet, answers to such questions are precisely what could provide an economic edge for one farm manager over another. Additionally, answers to questions such as those noted here will be needed as production agriculture increasingly attempts to take advantage of site-specific crop inputs in a precision agriculture setting.

To answer extended questions like those above, a well specified production function (i.e., yield response model) is needed, one that mathematically projects the changes in expected crop yield associated with different crop input levels. For generating the desired yield response model, actually, one for each of corn and soybean, we use the general procedure discussed in Kastens, Schmidt, and Dhuyvetter (2003). But, some explanation is in order before results are explored.

The yield models we consider are of the asymptotic plateau type. One reasonable specification for this type of yield model is

$$\begin{aligned} \mathit{Yield} = {} & \mathrm{B0} * \left(1 - \mathrm{B1} * \exp\left\{-\mathrm{B2} * \mathit{fertN}\right\}\right) * \\ & \left(1 - \mathrm{B3} * \exp\left\{-\mathrm{B4} * \mathit{fertP} - \mathrm{B5} * \mathit{STP}\right\}\right) * \\ & \left(1 - \mathrm{B6} * \exp\left\{-\mathrm{B7} * \mathit{fertK} - \mathrm{B8} * \mathit{STK} + \mathrm{B9} * \mathit{CEC}\right\}\right) * \\ & \mathrm{sig}\left(\mathrm{B10} + \mathrm{B11} * \mathit{PHtrim} - \mathrm{B12} * \mathit{PHtrim}^2\right) + e\ . \end{aligned} \tag{20}$$

In Equation [20], *Yield* is the expected corn yield in bu/acre conditional upon the causal factors specified on the right-hand side. The B expressions (called parameters or coefficients), B0,...,B12, are numerical constants that must be determined. B0 is the model's asymptotic yield plateau, which is the level of yield expected to be approached as all causal factors approach their yield-maximizing levels. The term exp denotes the exponential function. The term sig denotes the sigmoid or logistic function, where sig(x) = 1/(1 + exp[-x]), and which constrains all of its output on the [0,1] interval. With the exception of *PHtrim* and *e*, the variable names in the equation are self-explanatory and have the same units as already discussed earlier in the paper. *PHtrim* is the same as *soil pH*, only with all values above 6.1 set to 6.1. Doing that makes the expression sig(.) behave as a quadratic plateau with respect to *soil pH*, with the plateau reached at a *soil pH* of 6.1. That is, *soil pH* levels above 6.1 are prevented from increasing yields because they simply are assigned a value of 6.1. Notice that the N expression contains no measure of soil test N. That is because TSFR drives its N recommendation from *yield potential* and N credits associated with previous crops, rather than soil test N. If this same exercise were completed in lower-rainfall areas of the country, a measure of soil test N could easily be inserted in the model.

In Equation [20], the variable *e* is referred to as an error term, and is designed to take up the slack, so that the equality of the left- and right-hand sides of the equation is true at every data point of interest. Alternatively, *e* can be thought of as the difference between the actual yield at some data point and the model-predicted yield at the same data point. Notice that, given certain restrictions are placed on the B0-B12 parameters, each of the parenthetically enclosed terms on the right-hand side of Equation [20] is mathematically constructed so that it can take on values only between 0 and 1. Thus, if everything were "perfect" for crop production (and *e* = 0), the predicted yield would be the model's plateau, B0. The parametric constraints are as follows. B1, B3, and B6 are constrained to be between 0 and 1. All other parameters are constrained to be positive.

Once the parameters of Equation [20] are known, given fertilizer and lime prices, corn price, and a suitable relationship linking lime to *soil pH*, the optimal (profit-maximizing) level of fertilizer N, P, and K, and lime, can be mathematical determined using only the expressions on the right-hand side of the equation (typically, along with the assumption that the error term *e* equals 0). Normally, the parameters would be estimated with real-life data on crop yields, soil test levels, and fertilizer levels, using a procedure that minimizes only the sum of the squared prediction errors (i.e., the sum of the e^2

series).

On average, we believe that farm managers generally apply the correct (profit-maximizing) amounts of crop inputs. Further, we might assume that the TSFR recommendations are also correct in this manner. After all, if the researchers behind TSFR believed otherwise, they would not recommend the crop input amounts they do. In short, our goal is to end up with a model whose fertilizer recommendations, across numerous possible outcomes of soil test measures and crop yields, closely match those of TSFR, at least on average. Furthermore, we wish to end up with a model whose yield predictions are reasonably accurate, whenever the fertilizer and lime rates assigned in the yield model are the same as those the model recommends. That is, model-predicted yields when crop inputs are applied optimally ought to be close to observed yields.

Because TSFR's recommendations are believed suitable for a vast range of crop growing conditions in the tri-state region, we would like a data set for model estimation that also contains widely varying yield and soil fertility conditions. Real-life data sets of this magnitude are generally not available, and so we proceed with simulated data instead. To estimate the model, we use 10,000 simulated data points that we might expect to observed in the tri-state region. Each data point has an *STP* value, an *STK* value, a *soil pH* value, a *CEC* value, a corn yield value, and a soybean yield value. The *STP* variable is simulated around a mean of 20 and a standard deviation of 10 (the CV, or coefficient of variation, which is the standard deviation divided by the mean, is hence 0.5). *STP* is simulated as a log-normal variable. *STK*, also considered to be log-normal, uses a mean of 120 and a standard deviation of 60 (CV = 0.5). *CEC* is considered normal and uses a mean of 12 and a standard deviation of 3 (CV = 0.25). The *soil pH* variable is considered normal around a mean of 6.0 and a standard deviation of 0.375 (CV = 0.0625). Also, Polizotto suggested that *CEC* and *STK* probably are negatively correlated. Hence, we simulated our data to result in an arbitrarily-selected correlation between these two variables of &0.3. Finally, we simulated corn yield (as a normal variable) around a mean of 165 and a standard deviation of 41.25 (CV = 0.25), and soybean yield (also normal) around a mean of 50 and a standard deviation of 12.5 (CV = 0.25).

The model displayed as Equation [20] specifies principally decision variables (e.g., fertilizer and fertility) on the right-hand side, allowing all other causal factors to "fall into" the error term *e*. That means that, as far as the model goes, other unaccounted for factors are not allowed to impact the crop input decision, despite the fact that they often do in real-life crop input decision making. In fact, TSFR makes liberal use of a measure it calls *yield potential* (sometimes called yield goal), which is a value that the decision maker is expected to insert, and presumably accounts for some of these otherwise unaccounted for causal factors. Furthermore, we ought to be able to ask after-the-fact questions like, Had I known this year was going to be a drought (i.e., had I known what my yield goal should have been), how much fertilizer should I have applied?

To better capture the "variable yield goal" framework just described, using the simulated yield data, we create another variable, *Z*, equal to the simulated yield at each data point, divided by the maximum yield across all 10,000 yield values for a crop. This is exactly the procedure used in Kastens, Schmidt, and Dhuyvetter (2003), which is better described statistically there, but better described intuitively here. Notice that *Z* is like each of the parenthetical terms of Equation [20] in that it ranges from 0 to 1. Additionally, its expected value is 0.5. This transforms Equation [20] to the one we actually estimate:

$$\begin{aligned} Yield = {} & \mathrm{B0} * Z * \left(1-\mathrm{B1} * \exp\left\{-\mathrm{B2} * fertN\right\}\right) * \\ & \left(1-\mathrm{B3} * \exp\left\{-\mathrm{B4} * fertP - \mathrm{B5} * STP\right\}\right) * \\ & \left(1-\mathrm{B6} * \exp\left\{-\mathrm{B7} * fertK - \mathrm{B8} * STK + \mathrm{B9} * CEC\right\}\right) * \\ & \mathrm{sig}\left(\mathrm{B10} + \mathrm{B11} * PHtrim - \mathrm{B12} * PHtrim^2\right) + e\ . \end{aligned} \qquad [21]$$

A model like Equation [21] was estimated for each of corn and soybean, except that the soybean model lacked the N expression in the model. If one is more comfortable thinking about a model like Equation [20], rather than the *Z*-including one here, the B0 value estimated here need only be multiplied by 0.5 (the expected or average value of *Z*) to get back to a framework where *Z* is excluded.

In general, the parameters of each yield model were estimated in an iterative fashion, all the while trying to get model-recommended fertilizer rates "close" to the rates recommended by TSFR, and all the while trying to get model-predicted yields "close" to the yields simulated. That is, for a given iteration, the estimation algorithm assigns values to each of the B0-B12 parameters. Then, at each of the 10,000 data points, calculus is used to compute model-derived profit-maximizing fertilizer rates based on the B0-B12 parameter values and the value of each right-hand-side variable at that data point. That is, the profit-maximizing level of an input is computed by setting the derivative of yield with respect to the fertilizer variable of interest equal to the ratio of fertilizer price ($/lb) to crop price ($/bu). Because the optimal rate for one fertilizer depends on the optimal rate for each of the other fertilizers, a sub-routine was used to get all fertilizer values to converge on their optimal values at each data point for that iteration. During this sub-routine for computing optimal fertilizer rates, the *soil pH* term of Equation [21] was evaluated at both the *PHtrim* value shown, and at 6.1, and subsequently averaged. This assumes that *soil pH* likely will be halfway between these two values on average over time. Hence, because perfectly optimal lime rates are not computed, it is not strictly true that *all* other inputs are optimized whenever a given fertilizer's optimal rate is computed.

Following computation of optimal fertilizer rates, a measure of "closeness," (described later) is calculated relating each model-determined fertilizer value to the value recommended by TSFR given the same soil test conditions. Where needed by the TSFR formulas, expected yield is taken to be model-predicted yield, and *yield potential*, or yield goal (used in the N recommendation), is taken to be 1.1 times the expected yield. Plus, an accuracy measure for yield prediction is calculated as well. Then the estimation algorithm moves on to the next iteration. But, this is not quite the complete picture, as several other issues regarding TSFR's recommendations and our modeling efforts must first be resolved.

The economics of lime application in the model

In our modeling effort, we make the same assumptions made in the earlier section discussing rent premiums associated with different levels of *soil pH*. Namely, on average, we expect optimal *soil pH* to be 6.1, we expect to re-lime when *soil pH* drops below 5.8, and we expect that to happen in two years. Because of these assumptions, during the iterative model estimation procedure, *soil pH* was handled differently than N, P, and K. That is, at a given iteration and for each data point, the model's predicted yield was calculated for three different *soil pH* levels, 5.8, 5.95, and 6.1. The model-predicted yield difference between *soil pH* levels of 5.8 and 6.1 would be the expected yield return the first year following a lime application, where the lime was applied following harvest in the year before the yield gain. Because, *soil pH* is expected to drop to 5.95 for the second year or crop, the model-predicted yield difference between *soil pH* levels of 5.8 and 5.95 would be the expected yield return in the second year following a lime application. These two yield differences, after being assigned crop prices, and after being appropriately discounted in an NPV framework, comprise the dollar returns to a single lime application. Note that, although we simulate yields at three discreet *soil pH* levels for purposes of valuing a lime application, the yield model considers yield to be a continuous function of *soil pH* (at least below *soil pH* = 6.1). Also note that, like before, we consider the first crop in a sequence to be corn and the second crop soybean.

The cost required to acquire the dollar returns to a lime application depends on the amount of lime applied. Consequently, for the 5.8 *soil pH* level surmised at a data point prior to liming, we used *CEC* at that data point in the manner described earlier to derive the associated *buffer pH* level. Then, this was used along with Equation [13] to determine the amount of lime presumed applied to get *soil pH* back to 6.1. Then, the cost of this amount of lime was calculated by multiplying it by the lime price – with one exception. If the recommended lime rate was less than 1 T, we assumed the cost to be the same as the cost for 1 T. This is based on the idea that lime applicators charge by the ton with no separate application charge, but that they likely have a 1 T minimum. Finally, this lime cost, which requires no discounting since it occurs in year 0 of our NPV framework, is compared to the dollar returns to a single lime application discussed in the preceding paragraph – with the model estimation procedure attempting to get them equal at each data point. The goal of making them equal is based on the idea that marginal cost equals marginal revenue at the profit maximizing point. Furthermore, TSFR must expect that profits are maximized with liming upon a *soil pH* drop of 0.2 to 0.3 points. Otherwise a different drop would have been recommended. Of course, we might have used a different expected annual drop in *soil pH* in the absence of liming, but our assumption is consistent with information from Vyn and Polizotto, and hopefully consistent with the thinking of other decision makers in the area. Regardless, most of our comparisons for rent premiums are based on *differences* in crop input programs, so any inherent bias in our assumptions will mostly be netted out.

Special considerations for P and K

For P and K, TSFR clearly expects yield to respond to both fertilizer and fertility, with the two being substitutes for each other economically speaking. This is immediately obvious in TSFR's buildup recommendations. But, it is at least implicitly true even at much higher soil test levels, all the way up to the *CL3* level (which is 40 ppm for *STP*), above where no fertilizer is recommended. Of course, TSFR may not expect a response to fertilizer *this* year given sufficient soil test. But, a negative yield response is certainly expected the next year (or at least at some point in the future) in the absence of fertilizer this year. If this were not the case, TSFR would not recommend fertilizer all the way up to the *CL3* critical level. That is, we believe TSFR makes fertilizer recommendations that are expected to maximize farm profit. Consequently, in the yield model specified, it can be seen that we too allow yield to respond to both fertilizer and soil test for P and K, which is the same type of specification used in Kastens, Schmidt, and Dhuyvetter (2003).

Setting aside the idea that TSFR's recommendations are consistent with yield responding to both fertilizer and fertility in a substitute relationship, it is difficult to ignore TSFR's explicit belief in some long-run critical soil test level, which should be maintained into infinity with the application of crop removal fertilizer rates. Of course, the long-run critical levels purported by TSFR are not entirely consistent with an infinite time horizon because it would seem that there should be only one infinite-horizon critical level. That is, TSFR recommends a *CL1* long-run level for soils currently testing below *CL1*, a *CL2* long-run level for soils currently testing above *CL2*, and a long-run level equal to the observed soil test level when it is between *CL1* and *CL2*. Nonetheless, our model estimation procedure targets the TSFR-stated long-run critical levels in an infinite-horizon setting.

To obtain the model-derived long-run recommended levels for *STP* and *STK*, we consider that changing *STP* and changing *STK* has a cost equal to the cost of the amount of fertilizer P or fertilizer K needed to effect that change, namely the transformation rates described earlier (20 lb/acre of phosphate to increase *STP* by 1 ppm and [4 + 0.2 (*CEC*] lb/acre of potash to increase *STK* by 1 ppm). As described earlier, the profit-maximizing level of an input is computed by setting the derivative of yield with respect to the input equal to the price ratio of the input to the output (yield). But, in this case, the output is actually all future crop yields that arise from the increase in soil fertility, discounted back to the present. Hence, the "price," or denominator part of the input-to-output price ratio is actually the crop price multiplied by the infinite-horizon *DFAC* term discussed earlier, which is 20.8341 given our interest and tax rate assumptions. Then, evaluating the derivative at a fertilizer rate equal to crop removal (because that is the amount supposedly applied in a long-run situation), setting it equal to the stated price ratio, and solving for the relevant soil test level, results in a model-recommended optimal infinite-horizon soil test level. Then, the iterative model estimation algorithm tries to make this value for each data point close to the long-run critical level recommended by TSFR.

In summary, the parameters of Equation [21] were estimated iteratively for each of corn and soybean. For a given crop's model, conditional upon the 10,000 soil measures and the parameter values selected at each iteration, the 10,000 model-predicted yields were calculated. Next, the optimal fertilizer inputs were calculated (10,000 for each of N, P, and K; not N for soybean). Next, the 10,000 model-based lime expenditures were computed to take *soil pH* from 5.8 to 6.1 at each point. Finally, the model-determined long-run *STP* and long-run *STK* levels were computed in the manner just described. Next, at each iteration, these 70,000 measures (60,000 for soybean) were compared to the simulated yields or to the corresponding values recommended by TSFR, or to the calculated economic returns to lime, with an effort to make them as close as possible upon increasing iterations – until such point when no further gains in closeness are forthcoming.

In the model estimation process, it would be possible to use the squared difference (i.e., the error) between model- and TSFR-recommended values as our measure of closeness, which is essentially a minimizing the sum of squared errors (SSE) framework. But, given that the measures are scaled differently, it would be difficult to know how to weight the 7 individual components to the corn model SSE, for example. Consequently, we used a procedure called maximizing cross entropy (MCE).

In a simple 2-support maximum entropy (ME) framework, each estimated value must be assigned a lower and an upper bound (called supports). The estimate can then be expressed as some probability of the lower bound (p) and hence a probability of the upper bound of (1&p). The entropy function to be maximized would be H = &[p (ln(p)+(1&p) (ln(1&p)]. With no constraints, H is maximized when p = 0.50, which implies an estimate midway between the lower and upper support. MCE, on the other hand, allows for an estimate to have a "prior" that is different from the midpoint of the supports. Like the estimate itself, that prior can be expressed in terms of a probability of the lower bound, q. Now, the function to be maximized is G = &p (ln(p)&(1&p) (ln(1&p)+p (ln(q)+(1&p) (ln(1&q). For our model, the lower support for a series, for example, the *fertP* series of 10,000 fertilizer P values in the corn yield model, is taken to be 0.9 times the minimum of the 10,000 corresponding TSFR-recommended rates. Similarly, the upper support is 1.1 times the maximum of the TSFR recommendation series. The prior for the MCE framework for a particular estimate is taken to be the corresponding TSFR rate. Hence, each of the 70,000 estimates in the corn model has its own G value, and the estimation algorithm seeks to maximize the sum of all 70,000 G values by changing the B parameter values. Table 1 presents the final parameter estimates for the corn and soybean model derived using the MCE procedure, with the B0 values reported as the estimated B0 times 0.5, to make them compatible with the Equation [13] expression.

Table 1. Parameter estimates of yield models

corn model		soybean model	
parameter	value	parameter	value
B_0	193.8240	B_0	55.7569
B_1	0.9999	B_1	NA
B_2	0.0192	B_2	NA
B_3	0.9983	B_3	0.7732
B_4	0.0524	B_4	0.0663
B_5	0.0898	B_5	0.0758
B_6	0.9542	B_6	0.9958
B_7	0.3569	B_7	0.2773
B_8	0.1358	B_8	0.1283
B_9	2.3483	B_9	2.3785
B_{10}	-9.3505	B_{10}	-11.1337
B_{11}	2.1724	B_{11}	2.2187
B_{12}	-0.0526	B_{12}	-0.0055

Using a yield model to make inferences over time

It is generally straightforward to use an estimated yield model to answer the what-if questions needed. For example, one might ask, Conditional on soil test measures of interest, what will happen to yield if I increase my level of phosphate by 20 lb/acre from some other value? Then, along with crop prices and fertilizer prices, the question can be answered from a profitability standpoint. Next, since we have already described how *STP* and *STK* evolve over time given transformation rates, yields, and crop removal values, and since we have already presumed how *soil pH* might be expected to change over time in the absence of liming, we can ask how future yields might be impacted by crop inputs today via changes in soil fertility over time. With proper use of the NPV framework, these expected agronomic changes can be translated to expected economic changes.

One piece of the puzzle is still missing, and that regards expected changes in *soil pH* given we apply some lime rate different than that designed to reach a specific target. To obtain this, we first simulated 100 observations of *CEC* using 100 normal scores to ensure proper representation of each of the *CEC* levels we consider possible in this “small” series. This series of 100 *CEC* values is matched to each of

100 similarly-selected *soil pH* values, making 10,000 observations. Then, this data series is matched to each of the three targeted *soil pH* values (6.0, 6.5, and 6.8) covered by a lime recommendation equation in TSFR. Next, the *soil pH* values and the targeted *soil pH* values were used to compute a *proportional change in soil pH series*, which is simply (targeted *soil pH* &*soil pH*) divided by *soil pH*. Also, the three lime recommendation equations provided the necessary lime application rates given our assumed relationship between *buffer pH*, *soil pH*, and *CEC* as expressed in Equations [18] and [19]. Finally, the *proportional change in soil pH* series was modeled as the dependent variable in a simple statistical regression considering *lime*, *soil pH*, and *buffer pH* as explanatory variables, and using only the 79.7% of the observations where the change in *soil pH* was positive (i.e., where *soil pH* < targeted *soil pH*):

$$\mathit{proportional\ change\ in\ soil\ pH} = \mathrm{D1} * \mathit{lime} * \mathit{soil\ pH} + \mathrm{D2} * \mathit{lime} * \mathit{buffer\ pH}\ . \qquad [22]$$

Equation [22] was estimated without the usual intercept term to ensure that a 0 *lime* rate would predict no change in *soil pH*. The regression estimates were D1 = -0.037388 and D2 = 0.038382, and the model R-squared was 0.66. Now, with this model in hand, resultant changes in *soil pH* can be estimated from arbitrary lime amounts.

The what-if process just discussed can easily be turned into a profit-maximizing scenario using a suitable optimizing mathematical algorithm. Hence, one might ask for example, What are the profit-maximizing fertilizer and lime rates to apply over the 5 years of a rental contract? How does the profitability of this scenario compare to that of another, which starts from different soil test levels? That is, how much more or how much less rent can I pay in one scenario compared to another? How do changes in expected crop prices or crop input prices change the answer? How does a change in the length of the rental contract change the answer. Additionally, though we assume odd-numbered years are planted to corn and even-numbered years are planted to soybean, this framework easily would allow for a different sequence involving these two crops. Finally, our analysis brought a slightly higher level of economic reality to the problem by considering application charges. For example, N application was charged a rate of $6.00/acre, which essentially assumes it is NH_3. Then, if either of P or K is presumed applied, an application charge of $4.80/acre is assigned, which presumes that P and K are jointly applied. Similarly, if an optimal lime rate was selected that was less than 1 T/acre, it was assigned a total cost equivalent to 1 T. Such application-type charges prevent occasional simulated near-0 fertilizer and lime rates that would not be expected in the real world, precisely because there is an explicit or implicit fixed cost to application.

Results of the modeling exercise

This modeling exercise was undertaken partly to see if its added flexibility might provide different conclusions about rent premiums associated with soil fertility than the ones developed using the simpler framework earlier in the paper. That is, which of the features shown in or implied by earlier figures were preserved through this analysis and which ones have changed? Second, it was designed to help make improved site-specific crop input decisions, especially in a multi-year setting.

The first major question one might ask of this framework is, Does the model recommend fertilizer rates that are similar to those of TSFR? If not, why? It is difficult to know exactly what to expect in this regard. After all, the modeling exercise was designed to capture the economics believed explicit or implicit in the TSFR recommendations. But, possible economic inconsistencies in the TSFR recommendations might serve to "tug" the fertilizer/fertility/yield relationships unexpectedly. For example, one of the greatest possible inconsistencies in TSFR's recommendations (or perhaps our interpretation of them) is in the expected yield response to soil test measures (*STP* and *STK*) in the short run and long run, or, what is typically the same thing, at low soil test levels and high soil test levels. Our models attempted to mimic TSFR's long run soil test levels (the *CL1* and *CL2* values) by estimating the models so that their yield response to soil test levels would be appropriate given the cost of changing soil test levels. But, when that same responsiveness is coupled with fertilizer responsiveness, especially the high fertilizer responsiveness expected at low fertility levels, the projected yield might be too high to accurately reflect our simulated yields. Hence, something may have to "give" during model estimation.

To examine the differences between the model's recommendations (believed consistent with TSFR's recommendations) and those taken directly from TSFR, we consider a simulation exercise where one fertility or yield measure is varied while holding the other ones constant at their expected levels (and *CEC* =12). Then, optimal fertilizer rates were calculated in the same way they were calculated during the model estimation, which has already been described.

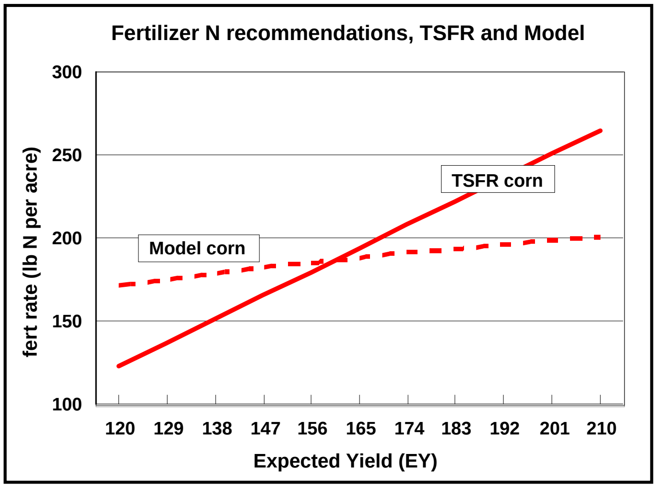

Figure 8

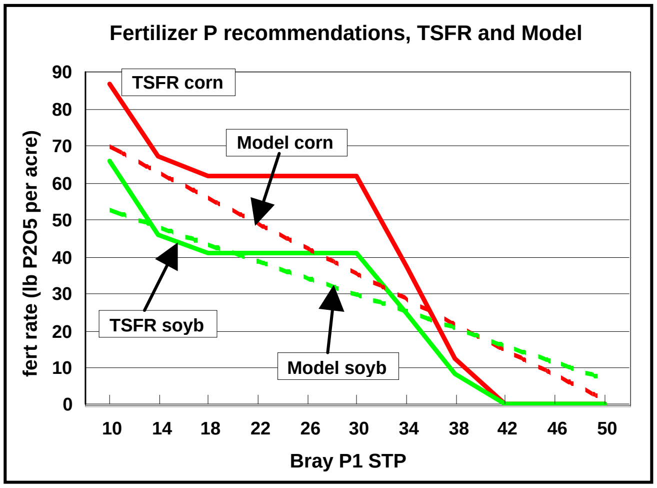

Figure 9

The first TSFR/model comparison is for N on corn and is shown in Figure 8. In this case, the N rates were quite similar at the mean yield (165 bu/acre) but the model's rates did not vary nearly as much as did TSFR's as expected corn yield varied. The next input of interest is P, with Figure 9 depicting TSFR- and model-recommended rates for different levels of *STP*. The model estimation process pulled P rates down from TSFR's recommendations, at least on average for corn. For soybean, the model and TSFR suggested similar rates on average. Long-run (infinite horizon) *STP* levels expected by TSFR and the model are shown in Figure 10. Since the model only allowed for a single long run *STP* level for a crop, it seemed to do a reasonable job of predicting a level close to the average of TSFR's suggested levels – at least for soybean. The corn long run *STP* level was slightly below that recommended for soybean. For the model to have predicted higher fertilizer rates at low *STP* levels (i.e., get them closer to TSFR's recommendations in Figure 9), it would have had to "pull" the long-run *STP* levels downwards (otherwise predicted yields may have been too far from their marks), thus further from TSFR's suggested long-run levels in Figure 10.

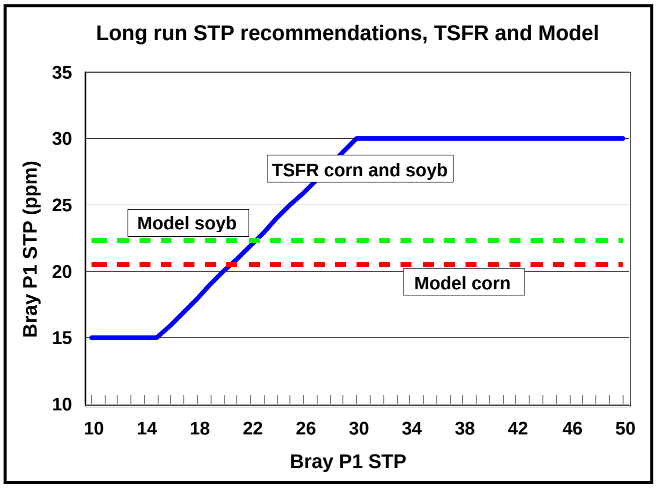

Figure 10

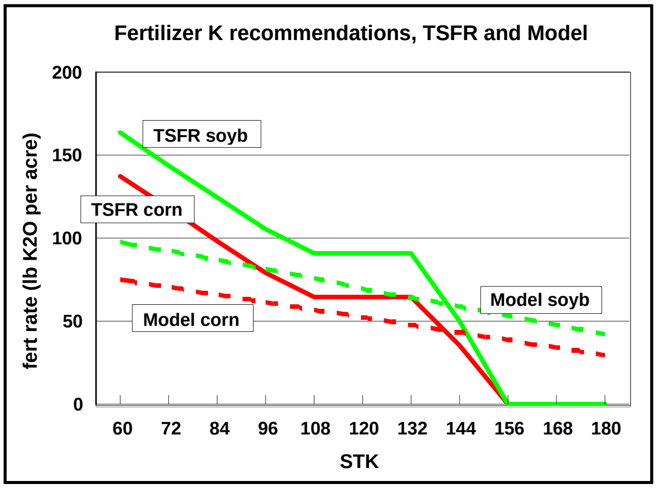

Figure 11

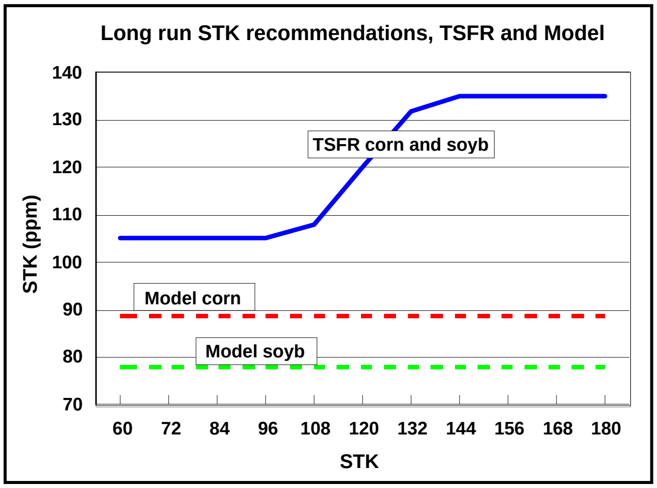

Figure 12

Figure 11 shows K fertilizer rates suggested by TSFR and the model. Figure 12 shows long-run *STK* levels suggested by TSFR and the model. Of the model-based figures shown, Figure 12 likely is the least believable since the model-recommended long-run *STK* levels were substantially below those of TSFR. Apparently, that is what the model had to "do" during estimation in order to balance the responsiveness to fertilizer and fertility in the case of K. Again, the potential long- and short-run inconsistency was most apparent. That is, increasing the yield responsiveness to *STK* to get long run levels closer to those of TSFR in Figure 12 would only have pulled the fertilizer rates further from their marks in Figure 11. Obviously, future work of this sort will need to rethink the short- and long-run implications of recommendations such as those from TSFR.

It is possible that the fertilizer recommendations of TSFR do not intend to reflect the responsiveness of yield to fertilizer. For example, the recommendations of a nearby state, Minnesota, conveyed at its website (www.extension.umn.edu) are closer to the recommendations of our models than to those of TSFR. As a direct example, Minnesota recommends 60 lb/acre of potash for an expected soybean yield of 50-59 bu/acre and *STK* = 41-80. If it is true that TSFR does not intend to reflect fertilizer responsiveness, rather only that of soil fertility, as noted before, TSFR has not adequately conveyed its reasons for suggesting a 4-year build. Obviously, continued research along these lines will need to depend heavily on trying to get a better understanding of what it is that TSFR is trying to optimize by its fertilizer recommendations.

The second major question to ask of our modeling framework is, How do rent premiums associated with soil fertility levels here compare to those from the simpler framework reported earlier? For this, we essentially will ignore the differences in model- and TSFR-recommended rates shown in the preceding figures, trusting that our models are reasonable enough depictions of reality that they can be used to study tradeoffs between fertilizer and fertility and profit over time.

The benchmark rent in this part of the analysis would be the one expected to occur in the presence of TSFR-stated optimal soil test conditions, at a typical *CEC* level, which we consider to be 12 meq/100g, and over an infinite horizon. The optimal soil test conditions are *STP* = 22.5 ppm, which is halfway between *STP*'s *CL1* and *CL2* values, *STK* = 120 ppm, which is halfway between *STK*'s *CL1* and *CL2* values based on *CEC* = 12, and *soil pH* = 6.1. This exercise is a direct maximization-of-discounted-profit-over-some-horizon simulation, where the optimization chooses fertilizer and lime values each year over the time horizon. As a reminder, application charges are considered here, and we assume odd-numbered years are planted to corn and even-numbered years are planted to soybean. Finally, to reduce computer time, we will show only time horizons through 12 years, and will show only selected soil fertility levels.

Figure 13 shows the rent premiums associated with different levels of *STP*. Comparing the high *STP* lines in this figure to Figure 2, the rent premiums shown here are not nearly as great at short horizons. And, they do not fall with increasing horizon as much either. Likely, as alluded to earlier, this is partly because our models tried to "smooth through" the various goals or apparent inconsistencies in TSFR's recommendations.

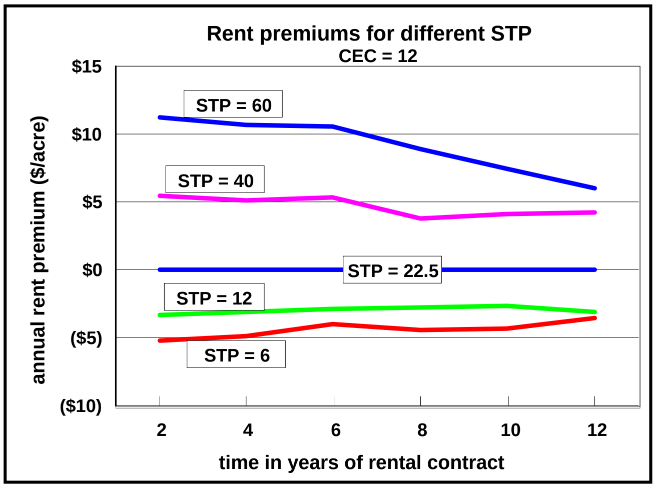

Figure 13

Figure 14 and Figure 15 show the model-determined rent premiums associated with different levels of *STK* and *soil pH*, respectively. As in Figure 13 with *STP*, the premiums shown here are not as great as in the simpler analysis earlier, especially at shorter horizons.

In the "fertilizer savings" rent premium framework developed earlier, we reckoned as reasonable the reported rent premiums associated with high soil fertility. Consequently, given the discrepancies between model- and TSFR-recommended fertilizer rates shown in Figures 8 through 12, we believe that the fertilizer savings approach may have better characterized rent premiums with high fertility than the model-based approach used here. On the other hand, we were unable to provide any useful information regarding rent discounts associated with low fertility using the fertilizer savings framework. Consequently, the low soil test lines of Figures 13 through 16 are probably a crude guess at what such rent discounts really ought to be. In particular, if the smoothing aspects of our modeling framework have understated the magnitude of the rent premiums associated with high fertility, it is probably also true that they have understated the magnitude of the rent discounts associated with low fertility (i.e., discounts probably should be greater than those shown).

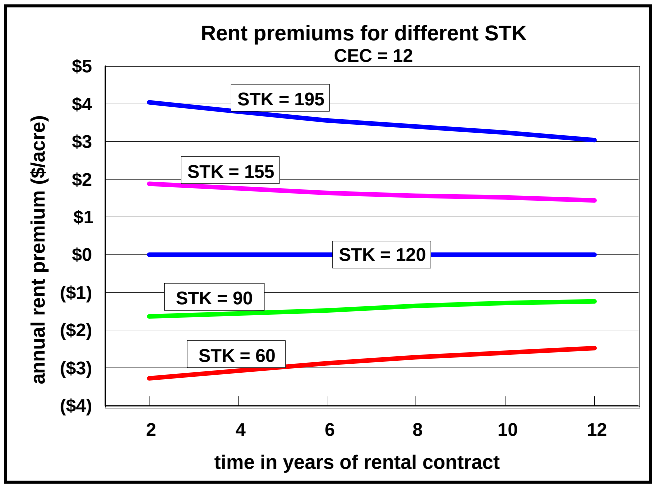

Figure 14

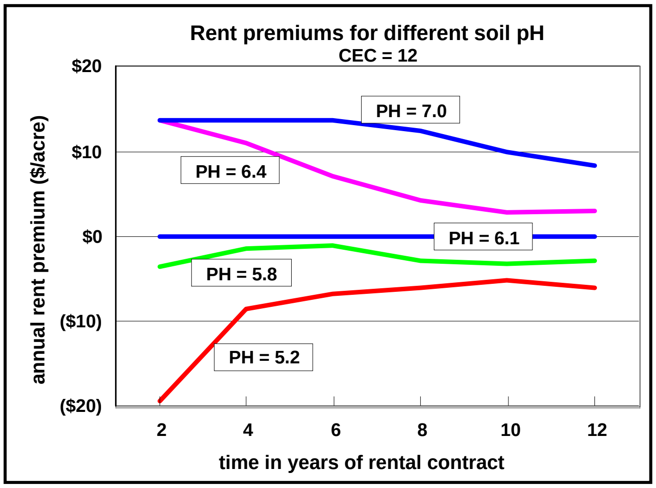

Figure 15

What might it take to construct a better yield model?

In 2002, Kansas State University (KSU) for the first time provided P and K fertilizer recommendations in a build/maintain/drawdown framework. But, in the process, KSU retained its earlier format, where fertilizer recommendations were based on a fertilizer-sufficiency framework. In a sufficiency framework, fertilizer is recommended up to a level where the last unit's cost is expected to just be covered by the value of the expected yield response associated with the last unit of fertilizer. Such fertilizer recommendations are appropriate for a 1-year horizon. That is, they would be correct for a tenant controlling land for only one year. For a yield modeling framework such as that used in this paper, sufficiency recommendations are precisely what is needed – because they accurately convey the expected relationship between yield response to fertilizer and yield response to fertility.

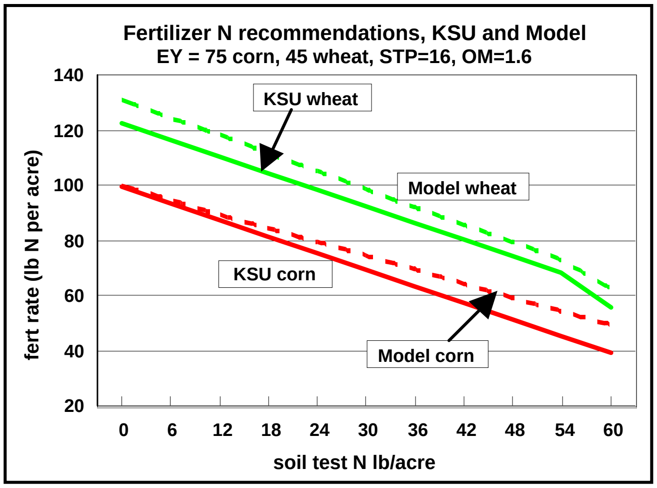

Figure 16

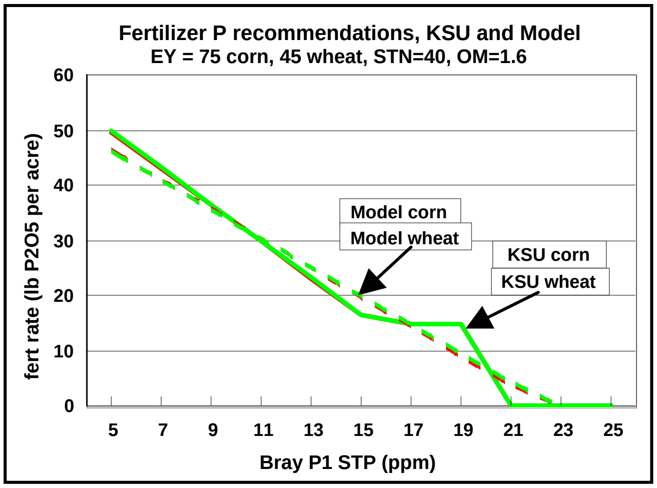

Figure 17

Conveying minimal details here, we used the modeling framework described earlier (at least those parts needed) to characterize wheat and corn yield response to P and N in a wheat-corn-fallow non-irrigated cropping program in northwest Kansas (expected yields are 45 bu/acre for wheat and 75 bu/acre for corn). Response to K and soil pH is not needed because area soils are high in these measures. Figure 16 shows that the models' fertilizer N recommendations were fairly close to those provided by KSU, at least in terms of N. Figure 17 conveys similar information for P. Given the expected yields considered, it turns out that KSU's sufficiency fertilizer P recommendations are nearly identical for wheat and corn. Consequently, the corn and wheat lines in the figure tend to be nearly on top of each other. As with N in Figure 16, the models' P recommendations in Figure 17 appear to match up quite well with KSU's P sufficiency recommendations. All in all, the yield models underlying Figures 16 and 17 appear to accurately reflect KSU's beliefs regarding yield response to N and P. Consequently, rental premiums and discounts associated with different levels of soil fertility simulated from these models should be considered highly reliable.

With *STP* now shown on the x-axis and horizon coming in as different lines in the figure, Figure 18 displays the rental premiums associated with different levels of *STP* across three different time horizons: 3 years, representing a short-term rental contract; 15 years, representing a more typical time period over which land is actually rented in Kansas by the same tenant and landlord; and 30 years, which is more typical of a land ownership situation.

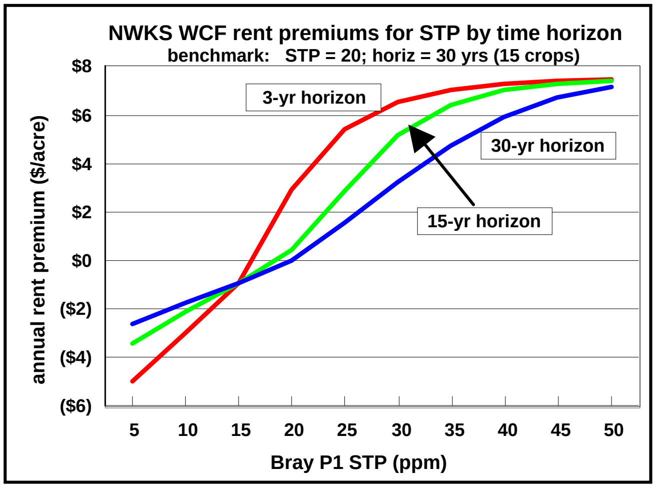

Figure 18

It is important to note that Figure 18's values should be interpreted in the light of annual cash rents in the area around $32/acre. In that regard, the premium associated with high levels of *STP* are quite substantial. For example, the rent premium associated with *STP* = 50 and a 3-year horizon is $7.45/acre, which is about 23% of cash rents in the area. On the other hand, Indiana 2003 cash rents on crop land average $103/acre (reported by USDA-NASS). The *STP* = 50, 3-year horizon, premium shown in Figure 2 was $15.97/acre, which is around 16% of cash rents in Indiana. Kansas *STP*-related rent premiums would contrast even more with those of Indiana if the Indiana recommendations were allowed to come from the model (i.e., those in Figure 13) rather than from the fertilizer savings approach shown in Figure 2 – at least if those premiums were calculated in a "percent of prevailing cash rents" manner. Thus, we believe that the Kansas results provide further evidence that the magnitude of the model-derived soil-fertility related rent premiums for Indiana are understated.

Summary

Crop input decision makers are becoming more aware of the capital nature of certain crop inputs, especially fertilizer (P and K) and lime. That fertilizer and lime can be treated as multi-year investments affords profit opportunities to those managers who better understand the fundamentals of investment analysis. Thus, this paper first provides a short tutorial on time-value-of-money analysis, referred to as NPV, for net present value.

We use the tri-state (Michigan, Ohio, and Indiana) fertilizer recommendations (TSFR), compiled by scientists at the three associated land grant universities (Vitosh, Johnson, and Mengel, 1995). Additionally, we draw from personal conversations with Kim Polizotto, agronomist for

PotashCorp/PCS Sales, and with Tony Vyn, extension agronomist at Purdue. The TSFR recommendations focus principally on critical fertility levels, where profit-maximizing producers are presumed to build up soils that are below critical levels by applying more fertilizer than crops remove. Similarly, high soil tests are associated with applying less fertilizer than the crop removes, so that soil tests can eventually fall back to critical levels. Then, soil levels at the critical levels were next compared to those with high fertility, to establish the economic advantage associated with different levels of fertility. The economic advantages were expressed in terms of rent premiums to a benchmark rent, where the benchmark was for land parcels with soil tests at critical levels. This analysis showed considerable rent premiums associated with high fertility, especially for short-term rental contracts.

A second analysis used the framework of Kastens, Schmidt, and Dhuyvetter (2003) to construct a mathematical yield response model. This model had several advantages over the simpler framework that had looked at only "fertilizer savings" to get at fertility-related rent premiums. In particular, the second analysis was needed to be able to draw any conclusions about rent discounts associated with low fertility.

The analyses provided here hinted at several possible inconsistencies in the way TSFR has developed its fertilizer recommendations. In particular, TSFR assumes a 4-year build of low-testing soils to their critical levels. If this buildup program implies a yield response to fertilizer and fertility in a substitute relationship, then the long-run critical soil test levels implied by this relationship are probably lower than those suggested by TSFR. On the other hand, if TSFR considers that yield responds only to soil test, where fertilizer is used merely to effect changes in soil test, then it is not clear why TSFR recommends specifically a 4-year buildup phase rather than an immediate one. We believe that these inconsistencies in TSFR's recommendations contributed to generating yield models that understated rental premiums and discounts that should be associated with different levels of soil fertility. As a contrast, we showed some results from a Kansas wheat and corn model generated from sufficiency recommendations for P. From those Kansas models, *STP*-related rent premiums and discounts appeared to be much larger than those of the comparable models developed for Indiana, at least when expressed as a percent of prevailing cash rents.

Our modeling approach that mathematically generalizes a provider's fertilizer recommendations into a yield model has a number of advantages besides providing potentially more accurate rent premiums and discounts associated with levels of fertility. In particular, it allows for consideration of crop and fertilizer prices in the fertilizer decision. Further, it could be expanded to include variables other than those used directly in a provider's fertilizer recommendations. Also, through simulation of reasonable spatially dependent data, it could be used to devise optimal soil sampling routines. Consequently, considerably more research in this area is needed, along with a better understanding of providers' fertilizer recommendations.

References

Kastens, T.L., J.P. Schmidt, and K.C. Dhuyvetter. 2003. "Yield Models Implied by Traditional Fertilizer Recommendations and a Framework for Including Non-traditional Information." *Soil Science Society of America Journal*. 67,1:(January-February 2003):351-364.

Vitosh, M.L., J.W. Johnson, and D.B. Mengel. 1995. "Tri-state Fertilizer Recommendations for Corn, Soybeans, Wheat, and Alfalfa." Extension Bulletin E-2567 (New), July 1995. Michigan State University.